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A Comparison of Static Bed and Moving Bed Chromatography

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Summary

An analysis of static and moving bed perfect column chromatography is carried out for the separation of a binary mixture having constant separation factor isotherms. It is shown that the carrier requirements are identical for both methods of operation. The model does not provide information on the length of the moving bed column, but shows that the length of the static bed column can be made vanishingly small by reducing the duration of feed intervals. Furthermore, separating the feed bands only partially can lead to a substantial reduction in the length of the static column at fixed separation rates.

INTRODUCTION

In the design and analysis of industrial chromatographic separation, the question is often posed as to whether there are any advantages to be gained in moving the column packing countercurrently to the fluid phase, and whether the simplicity of static column packing is not gained by sacrifice in efficiency. This paper attempts to answer these questions for the limiting case of a separation that is conducted in a perfectly efficient column. That is to say, only material balances and equilibrium relations govern the separation; the "kinetics" are extremely rapid. Furthermore, the equilibrium relations are of the constant "separation factor" type. This special case is interesting for several reasons:

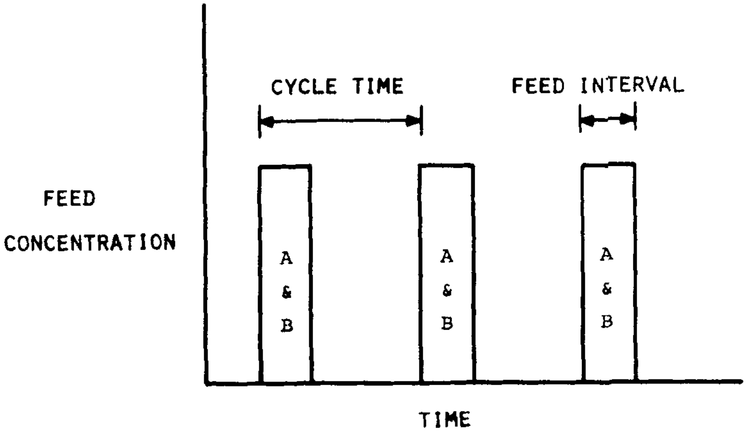


FIG. 1. Feed concentration at column inlet.

(a) These are important separations that are governed approximately by constant separation factor equilibrium. (b) This special case sets an upper limit on the performance of the real separation systems. (c) This limiting case displays the essential differences between the static and moving bed systems without becoming involved in difficult kinetic models.

The operation of the chromatographic separation is illustrated for the static column packing (bed) separating only two components. Figure 1 shows the variation of feed (A plus B) concentration at the

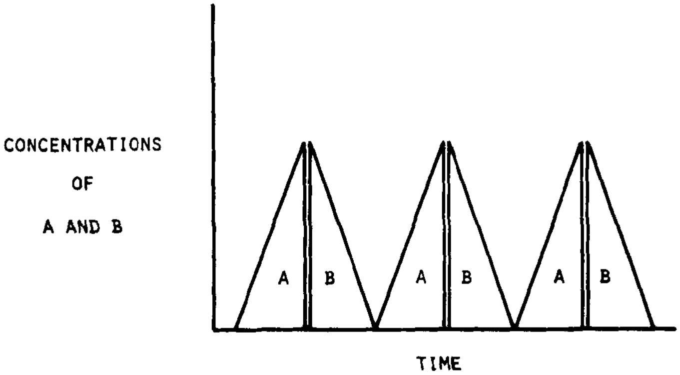


FIG. 2. Component concentrations at column outlet.

column inlet. The outlet concentrations of A and B would appear somewhat as shown in Fig. 2. In the case shown, the injections are spaced so that the B band from a succeeding injection just catches up with the A band of the preceeding injection at the column outlet. This is clearly a reasonably efficient way of operating the system.

The construction of the moving bed analog of the above process may be understood with the aid of Fig. 3, which shows the form of a partly separated feed band in the interior of a static bed column.

The basic "saw-tooth" form results from the assumption that the carrier fluid is retained by the column packing with an affinity intermediate between the affinities of A and B. It is, of course, not necessary that the carrier have such an affinity for the packing but this choice, as opposed to the choice of a more weakly or more strongly retained carrier, appears to reduce carrier requirements and the column volume needed.

The portion of the column occupied by A and B consists of three functional sections, labeled Section 2, Section 3 and Section 4/1. (The reason for the 4/1 numbering will become clearly shortly.) In Section 3, the carrier displaces the more strongly adsorbed Component A. In Section 2, the weakly adsorbed Component B displaces the carrier. In Section 4/1, the separation between A and B takes place.

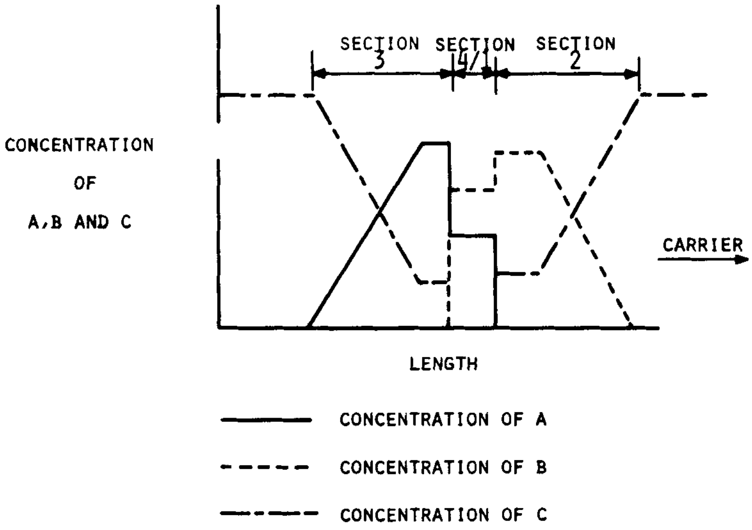


Fig. 3. Partly separated feed band within static bed column.

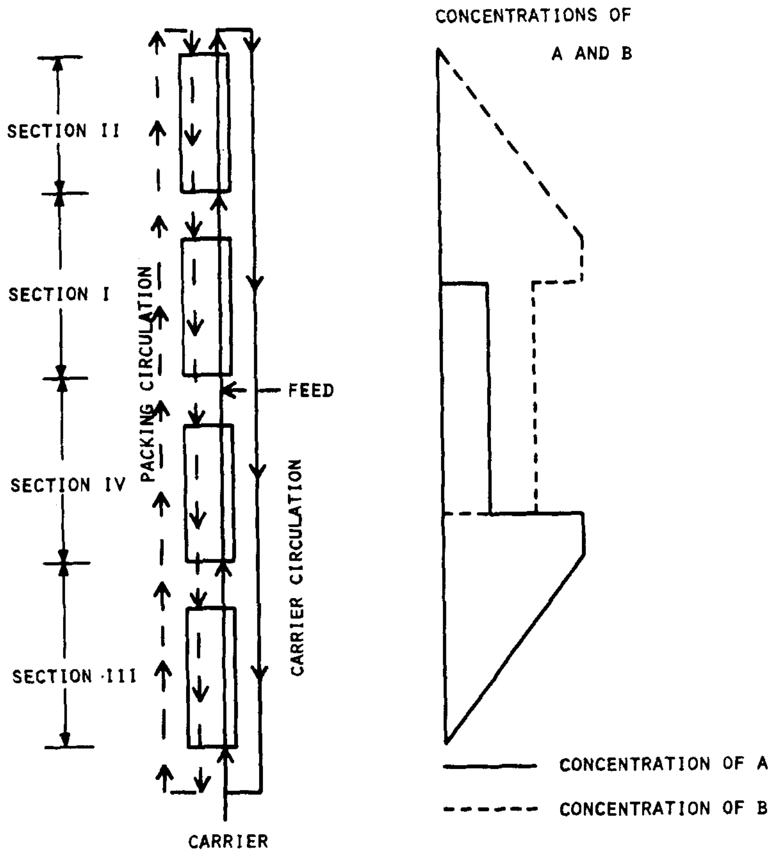


Fig. 4. Moving bed chromatography process.

The analogous moving bed process also consists of the above functional sections. For steady-state operation, Fig. 4 shows a schematic flow diagram of the system and one form of the associated concentration profiles.

Although the carrier enters at only one point of the system (bottom of Section III) and the feed only at the point between Section IV and Section I; the separated components A and B, along with the associated carrier C, are removed along the entire length of Sections III and II, and along a portion of Sections IV and I. It will be shown later that only by means of this continuous removal of separated components can steady-state operation be achieved.

Two key comparisons between the static and moving bed processes are: (a) Which process requires the least carrier fluid for a given amount of A and B separated? (b) Which process requires the least inventory of packing for a given rate of separation?

The answers are obtained by applying the following material balances and equilibrium relations to the two processes.

EQUILIBRIA

The capacity of the column packing for A + B + C is fixed at a certain saturation value, and the packing is always saturated. A fully ionized ion exchange resin in contact with a mixed salt solution of constant concentration satisfies this model very well. A molecular sieve whose pores are accessible to A, B, and C is not a bad example of such a system. The concentration of A + B + C in all portions of the system is constant.

The concentrations will be expressed in terms of total column volume. The concentration of A, for example, in the fluid phase is measured in (amount of A in fluid)/(volume of column) rather than (amount of A in fluid)/(volume of fluid). The concentration of A in the packing is likewise expressed as (amount of A in packing)/(volume of column).

By using these concentration units, three "constant separation factor" equilibrium relations obtain among the three possible pairs of components:

for A in presence of B

$$\frac{N_1}{N_0} = \frac{\alpha_{12}(n_1/n_0)}{1 + (\alpha_{12} - 1)(n_1/n_0)} \tag{1a}$$

for A in presence of C

$$\frac{N_1}{N_0} = \frac{\alpha_{13}(n_1/n_0)}{1 + (\alpha_{13} - 1)(n_1/n_0)} \tag{1b}$$

for B in presence of C

$$\frac{N_2}{N_0} = \frac{\alpha_{23}(n_2/n_0)}{1 + (\alpha_{23} - 1)(n_2/n_0)} \tag{1c}$$

It is possible to formulate simple extensions of these relations which hold when all these components are present simultaneously. These will not be presented here because, in the analysis of the moving and static bed systems, the simultaneous presence of all three components will not occur. Equivalent equilibrium relations occur frequently in the chromatographic literature (*1*).

MATERIAL BALANCES

The following material balance equation is derived by the usual procedure of considering an infinitesmal column slice shown in Fig. 5:

$$U_S(\partial N/\partial x) - U_L(\partial n/\partial x) = (\partial N/\partial t) + (\partial n/\partial t) \qquad (2)$$

Since no more than two components are present at any point in the column, N is related to n by Eq. (1), written for brevity as

$$N = f(n) \qquad (1d)$$

On combining Eqs. (2) and (1d):

$$(\partial n/\partial t) + [(U_L - U_S f'(n))/1 + f'(n)](\partial n/\partial x) = 0 \qquad (3)$$

When the column packing is stationary, Eq. (3) reduces to a form that has been investigated very completely in the chromatographic literature (*2–5*). The Rachinskii monograph (*5*) has been particularly influential in the development that follows.

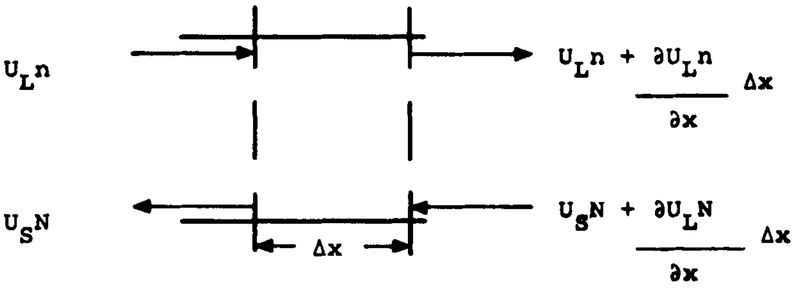


FIG. 5. Derivation of material balance.

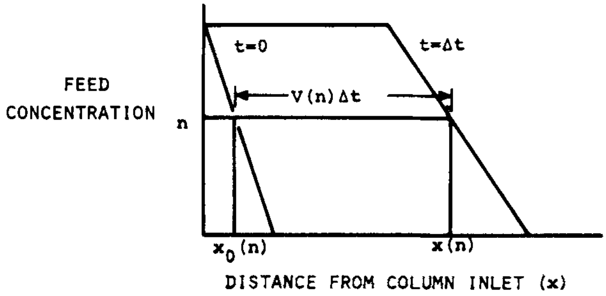


FIG. 6. Illustration of continuous solution.

STATIC COLUMN PACKING

Two types of solutions exist for this system, a discontinuous one and a continuous one. The discontinuous solution obtains when the time and distance derivatives in Eq. (3) are zero.

When a band of weakly retained component is introduced into a column containing only one, more strongly retained component, the following continuous solution can be shown to hold (*6*), illustrated in Fig. 6:

$$x(n) = V(n)t + x_0(n) \tag{4}$$

where

$$V(n) = U_L/[1 + f'(n)] \tag{5}$$

$V(n)$ is the velocity with which the concentration, n, moves down the column. For constant separation factor isotherms:

$$f'(n) = \frac{N_0}{n_0} \frac{\alpha}{[1 + (\alpha - 1)(n/n_0)]^2} \tag{6}$$

If α is less than 1, then

$$V(\bar{n}) > V(\bar{\bar{n}}) \qquad \text{when } \bar{n} < \bar{\bar{n}}$$

This case corresponds to the situation illustrated in Fig. 6.

The first, discontinuous, solution is obtained when $\alpha > 1$, since from Eq. (6), one obtains

$$V(\bar{n}) < V(\bar{\bar{n}}) \qquad \text{when } \bar{n} < \bar{\bar{n}}$$

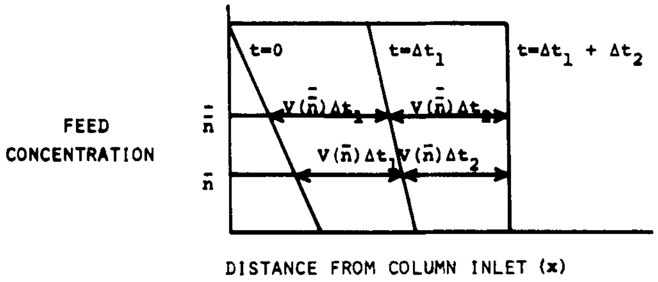


Fig. 7. Illustration of discontinuous solution.

Therefore, an initially sloping front eventually deteriorates into a discontinuous step, as illustrated in Fig. 7.

These solutions may be applied to the analysis of the following two part chromatographic cycle: (a) A mixture of the components to be separated is introduced for a period t_F into the column; the front portion of which is filled with carrier. (b) At the conclusion of the feed introduction portion of the cycle, carrier is again introduced into the column for a period t_C.

The concentration profiles at the conclusion of the feed introduction period are shown in Fig. 8.

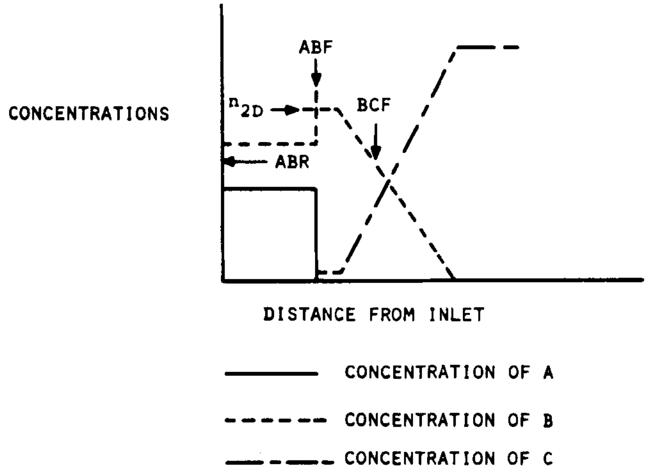


Fig. 8. Concentration profiles after feed introduction.

The velocity of the discontinuous front, ABF, is obtained from a material balance (Appendix A) on component A as

$$V_{ABF} = \frac{U_L}{1 + (N_1/n_1)} \tag{7}$$

$$\frac{N_1}{n_1} = \frac{N_0}{n_0} \frac{\alpha_{12}}{1 + (\alpha_{12} - 1)(1 - \phi)} \tag{8}$$

and ϕ is the fraction of component B in the feed:

$$\phi = n_{2F}/n_0 \tag{9}$$

Therefore,

$$V_{ABF} = \frac{U_L}{1 + (N_0/n_0)[\alpha_{12}/(1 + (\alpha_{12} - 1)(1 - \phi))]} \tag{10}$$

The velocity of the continuous front (BCF) is obtained from Eqs. (5), (6), and (1c):

$$V_{BCF} = \frac{U_L}{1 + \dfrac{N_0}{n_0} \dfrac{\alpha_{23}}{[1 + (\alpha_{23} - 1)(n_2/n_0)]^2}} \tag{11}$$

The concentration of Component B, in the flat portion of the B profile, between the ABF front and the BCF front can be obtained by a material balance on Component B, as shown in Appendix B.

Before A and B have become completely separated, the feed band has the shape shown in Fig. 9.

The velocity of the ABR front, V_{ABR}, is derived by the method of Appendix A:

$$V_{ABR} = \frac{U_L}{1 + (N_0/n_0)[1/(\alpha_{12} + (1 - \alpha_{12})\phi)]} \tag{12}$$

The concentration of Component A between ABR and ACR is derived by the method of Appendix B; and is given implicitly by

$$\frac{n_{1D}}{n_0}\left[\frac{\alpha_{13}}{1 + (\alpha_{13} - 1)(n_{1D}/n_0)} - \frac{1}{\alpha_{12} + (1 - \alpha_{12})\phi}\right]$$

$$= (1 - \phi)\left[\frac{\alpha_{12}}{1 + (\alpha_{12} - 1)(1 - \phi)} - \frac{1}{\alpha_{12} + (1 - \alpha_{12})\phi}\right] \tag{13}$$

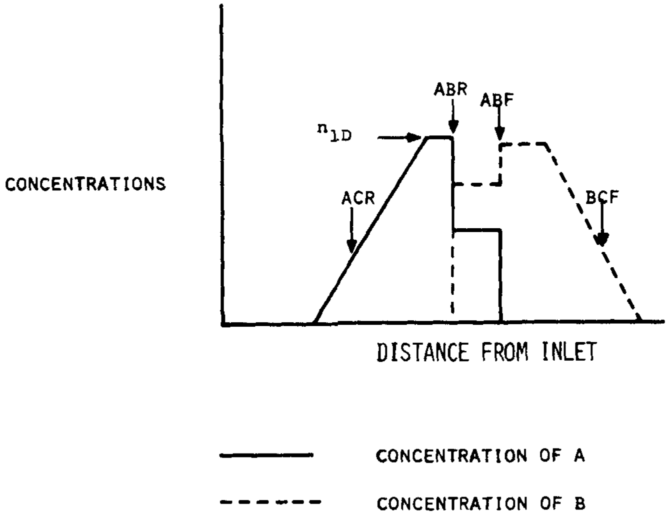


FIG. 9. Concentration profiles during feed band displacement.

The velocity of the ACR front is obtained from Eqs. (5), (6), and (1b):

$$V_{ACR} = \frac{U_L}{1 + \dfrac{N_0}{n_0} \dfrac{\alpha_{13}}{[\alpha_{13} + (1 - \alpha_{13})(n_3/n_0)]^2}} \qquad (14)$$

The time from the conclusion of the feed introduction period required to separate A from B completely, t_D, is determined by the condition

$$V_{ABF}\,(t_F + t_D) = V_{ABR}\,t_D \qquad (15)$$

The distance from the column inlet at which this superposition of the fronts occurs is the minimum column length for complete separation. It is

$$L_M = V_{ABR}\,t_D \qquad (16)$$

Combining Eqs. (16), (15), (12), and (10), one obtains

$$L_M = U_L t_F (n_0/N_0)[(1 + (\alpha_{12} - 1)(1 - \phi))/(\alpha_{12} - 1)] \qquad (17)$$

This equation shows that: (a) The column length can be made vanishingly small by using very short feed intervals. (b) Only one separation factor, α_{12}, influences the minimum column length. (c) The saturation capacity of the column packing should be large relative to the columns void volume.

If one operates the process so that the beginning of the B band of a succeeding injection just catches up with the end of the A band from the preceding injection, then the period t_C is given by

$$t_C = t_F[(1 + (\alpha_{12} - 1)(1 - \phi))/(\alpha_{12} - 1)](\alpha_{13} - \alpha_{23}) \tag{18}$$

The ratio of carrier to feed is simply the ratio of t_C to t_F, or

$$R_D = [(\alpha_{12}/(\alpha_{12} - 1)) - \phi](\alpha_{13} - \alpha_{23}) \tag{19}$$

This ratio does not depend on the length of the feed interval, but does depend on the difference of the separation factors of the feed components with respect to the carrier.

Another useful ratio is that of column volume to the rate of separation, R_C. This ratio will be computed for a somewhat more general method of operating the column; the column length provided will be shorter than L_M. As a result the portion of the feed band between ABR and ABF will be unseparated, and may be recycled.

If the column length, L, is $L = \beta L_M$, where

$$0 < \beta < 1 \tag{20}$$

it can be shown that

$$R_C = \frac{t_F}{N_0} \frac{1 + (\alpha_{12} - 1)(1 - \phi)}{(\alpha_{12} - 1)} \times \left[1 + \beta \frac{1 + (\alpha_{12} - 1)(1 - \phi)}{(\alpha_{12} - 1)} (\alpha_{13} - \alpha_{23})\right] \tag{21}$$

The column volume is minimized by making β small; i.e., by using a short column and recycling a large portion of the feed band. For reasonable values of the various separation factor, the coefficient term of β is about 1. Therefore, about a 50% reduction in column volume can be achieved by recycling almost all the feed. It can also be shown that the ratio R_D is independent of the amount of recycle, and therefore it is still given by Eq. (19).

Several conclusions may be drawn from Eqs. (19) and (21): (a) The separation factor α_{12} should be large in order to minimize carrier and packing requirements. But for feed mixture containing about equal quantities of A and B, values of α_{12} above 4, or so, do not lead to substantial further improvements. (b) The carrier requirements are sensitive to the difference, $\alpha_{13} - \alpha_{23}$. Therefore, the carrier must not be retained too strongly or too weakly by the column packing. (c) The saturation capacity of the column packing has an important influence on the packing requirements, but not on the carrier requirements.

MOVING COLUMN PACKING

Equation (3), applied to a countercurrent column operating at steady state, reduces to

$$[(U_L - U_S f'(n))/1 + f'(n)](\partial n/\partial x) = 0 \qquad (22)$$

Two types of solution exist. The first corresponds to the partial distance derivative being zero. This is associated with flat concentration profiles separated by discontinuous fronts. The second type corresponds to

$$U_L/U_S = f'(n) \qquad (23)$$

This shows that in a region of continuous variation of composition with distance and nonconstant $f'(n)$, there must be continuous variation in the ratio of the two phase velocities. In this exposition the solids velocity is assumed constant throughout the column, and fluid velocity is varied to achieve steady state.

The applicability of these two solutions can be determined by the following stability analysis of concentration profiles.

In Appendix C it is shown that the velocity ratio required for a stationary discontinuous boundary between two flat profiles is given by

$$\frac{U_{LD}}{U_S} = \frac{N_0}{n_0}\alpha\frac{1}{[1 + (\alpha - 1)(n_A/n_0)][1 + (\alpha - 1)(n_B/n_0)]} \qquad (24)$$

For the continuous front:

$$\frac{U_{LC}}{U_S} = \frac{N_0}{n_0}\alpha\frac{1}{[1 + (\alpha - 1)(n/n_0)][1 + (\alpha - 1)(n/n_0)]} \qquad (25)$$

Consider a discontinuous front in Section II of the system shown in Fig. 4. For Section II, Eqs. (24) and (25) refer to Component B, and the α is α_{23}, which is less than 1. The initial situation and a perturbation at Δt, which produces a continuous variation in n_2, are shown in Fig. 10. The problem is to determine whether this perturbation will be stable,

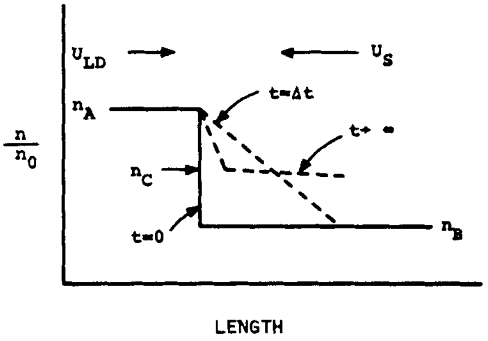


FIG. 10. Discontinuous front in Section II.

will propagate, or will be eliminated. The fluid velocity is held at U_{LD}, calculated from Eq. (24). It is apparent from Eqs. (24) and (25) that there exists a value of n_2, n_C: $n_B < n_C < n_A$ such that $U_{LC}(n_C) = U_{LD}$. But for $n_B < n_2 < n_C$, $U_{LC}(n_2) < U_{LD}$. And for $n_C < n_2 < n_A$, $U_{LC}(n_2) > U_{LD}$.

Thus, for the values of $n_2 > n_C$, the fluid velocity U_{LC} required for the stability of that concentration point is higher than the actual velocity, U_{LD}. Therefore, the perturbation will be "swept back" by the solids and the discontinuous front will be re-established. But, for the values of $n_2 < n_C$, the fluid velocity U_{LC} required for the stability of n_2 is less than the actual velocity, U_{LD}.

The perturbation will therefore propagate to the right. The profile will tend to develop as shown. In the limit, the concentration n_C replaces n_B as the right-side plateau concentration. The above reasoning applied

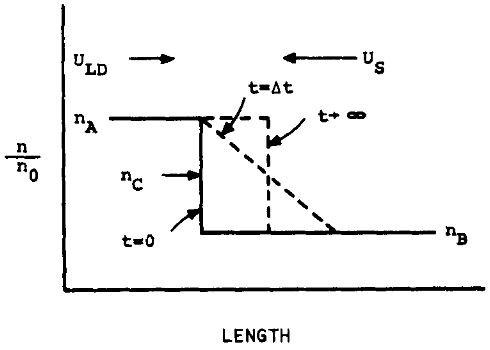


FIG. 11. Discontinuous front in Section IV.

to n_A, n_C, and a new $\bar{n}_C$ shows that this profile is also unstable. Thus, discontinuous profiles are unstable in Section II. This argument also proves that discontinuous profiles are unstable in Section III and therefore the second, continuous solution applies to Sections II and III.

In Section IV, Eqs. (24) and (25) refer to Component A, and the α is α_{12}, which is greater than 1. In Fig. 11, the initial discontinuous profile and its development are illustrated.

Again, it is possible to find an n_C for which $U_{LC}(n_C) = U_{LD}$. But for $n_1 > n_C$, $U_{LC}(n_1) < U_{LD}$. For $n_1 < n_C$, $U_{LC}(n_1) > U_{LD}$. The profile therefore tends to return to the discontinuous shape.

This also applies in Section I. Therefore the discontinuous solution applies to Sections I and IV.

The detailed analysis of the system proceeds as follows.

Section I

(Fig. 12)

Component A must be absent from the upper portion in order to produce pure B. Therefore, Eq. (24) when applied to Component A, is

$$U_{LI} = U_S(N_0/n_0)[\alpha_{12}/(1 + (\alpha_{12} - 1)(n_{1I}/n_0))] \qquad (26)$$

Section II

(Fig. 13)

The solids velocity, U_S, is the same as in Section I. The fluid velocity is varied by drawing off side streams according to Eq. (25).

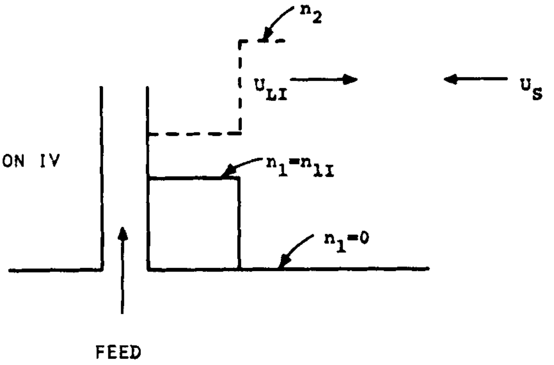


FIG. 12. Concentration profiles in Section I.

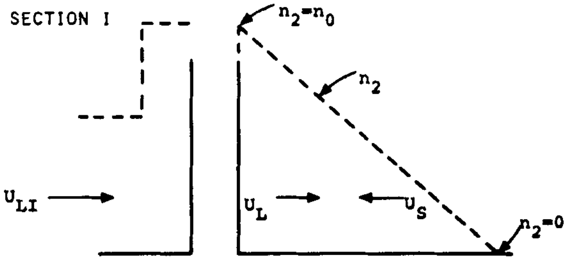


FIG. 13. Concentration profiles in Section II.

At the top of the column, $n_2 = 0$. This is necessary for the recirculating fluid not to contaminate Section III with Component B. At this point

$$U_L(0) = U_S(N_0/n_0)\alpha_{23} \tag{27}$$

At the bottom of Section II, $n_2 = n_0$ and

$$U_L(n_0) = U_S(N_0/n_0)(1/\alpha_{23}) \tag{28}$$

The amount of carrier Q_{CII} leaving with Component B is determined by a material balance on the carrier around Section II by using the end velocities calculated above:

$$Q_{CII} = U_S N_0 - U_L(0)n_0 = U_S N_0(1 - \alpha_{23}) \tag{29}$$

By a Component B material balance on the section of the system between the bottom of Section IV and the front of Component A in Section I,

$$U_{LI}n_0 - U_S N_0 = U_F \phi n_0 \tag{30}$$

where $U_F \phi n_0$ is the amount of Component B entering with the feed.

Section IV

(Fig. 14)

By an analysis similar to that of Section I, it is shown that

$$U_{LIV} = U_S(N_0/n_0)[\alpha_{21}/(1 + (\alpha_{21} - 1)(n_{2IV}/n_0))] \tag{31}$$

where $\alpha_{21} = 1/\alpha_{12}$.

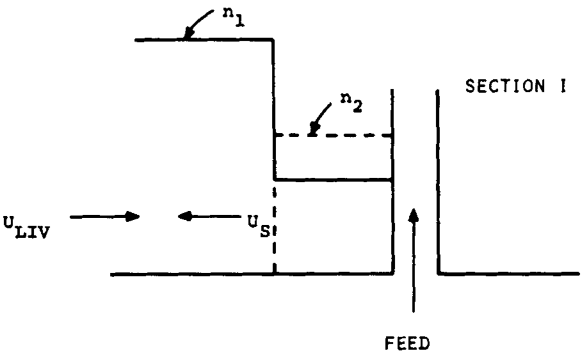


FIG. 14. Concentration profiles in Section IV.

Section III

(Fig. 15)

By an analysis similar to that of Section II, it is shown that

$$U_{LIII}(0) = U_S(N_0/n_0)\alpha_{13} \tag{32}$$

$$U_{LIII}(n_0) = U_S(N_0/n_0)(1/\alpha_{13}) \tag{33}$$

The carrier leaving with Component A is

$$Q_{CIII} = U_L(0)n_0 - U_SN_0 = U_SN_0(\alpha_{13} - 1) \tag{34}$$

By a Component A material balance,

$$U_SN_0 - U_{LIV}n_0 = U_F(n_0 - \phi n_0) \tag{35}$$

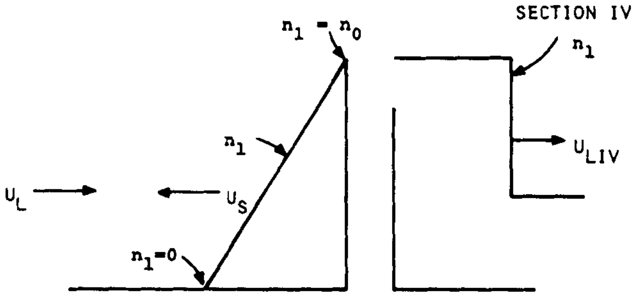


FIG. 15. Concentration profiles in Section III.

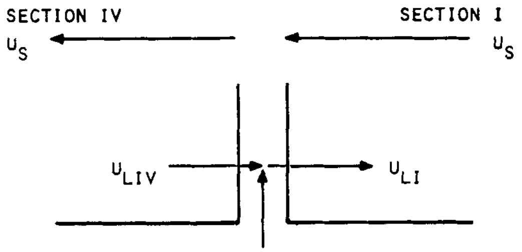


FIG. 16. Mixing point material balance.

Feed Point

(Fig. 16)

From a total and a Component A material balance, one obtains:

$$U_{LI} - U_{LIV} = U_F \tag{36}$$

$$U_{LI}n_{1I} - U_{LIV}n_{IV} = U_F(n_0 - \phi n_0) \tag{37}$$

Combining Eq. (37) with Eqs. (30), (35), (26), and (31), one obtains:

$$U_S/U_F = (n_0/N_0)[(\alpha_{12}/(\alpha_{12} - 1)) - \phi] \tag{38}$$

This very important equation relates the rate of solids circulation to the feed rate.

One also obtains the fluid velocities in Sections I and IV:

$$U_{LI} = U_F(\alpha_{12}/\alpha_{12} - 1) \tag{39}$$

$$U_{LIV} = U_F(1/\alpha_{12} - 1) \tag{40}$$

The concentration of Component A in the bottom of Section I is

$$n_{1I}/n_0 = 1 - \phi \tag{41}$$

which is its concentration in the feed.

The concentration of Component B in the top of Section IV is

$$n_{2IV}/n_0 = \phi \tag{42}$$

which is its concentration in the feed.

Adding Eqs. (29) and (34), one obtains the total amount of carrier leaving with the purified Components A and B as

$$Q_C = U_S N_0(\alpha_{13} - \alpha_{23})$$

The ratio of carrier to feed is then given by

$$R_D = [(\alpha_{12}/(\alpha_{12} - 1)) - \phi](\alpha_{13} - \alpha_{23}) \tag{43}$$

This is identical with the result obtained for the static bed column, Eq. (19).

A point that has been glossed over so far is the problem of matching velocities between Sections I and II and Sections III and IV.

The fluid velocity at the discontinuity in Section I is only a function of α_{12} and the feed composition (Eqs. 26 and 38). The fluid velocity at the bottom of Section II depends on α_{23} as well as on α_{12} and the feed composition (Eqs. 28 and 38). In general, the fluid velocity at the discontinuity of Section I will therefore not equal the fluid velocity at the bottom of Section I.

If U_{LI} is greater than $U_{LII}(n_0)$, then in order to equalize the fluid velocity at the top of Section I and the fluid velocity at the bottom of Section II, some Component B must be removed between the discontinuity in Section I and the bottom of Section II. Similarly, pure Component A may have to be removed between the discontinuity in Section IV and the top of Section III.

If, however, the velocity at the discontinuity in Section I is lower than the velocity at the bottom of Section II, one can avoid adding pure B. However, because of this mismatch in velocities, some carrier will work its way into Section I, and the concentration profiles in this section will be as shown in Fig. 17. The highest point reached by the Component B concentration, n_B, is such that

$$U_{LII}(n_B) = U_{LI}$$

The natural division between Sections I and II has disappeared, and

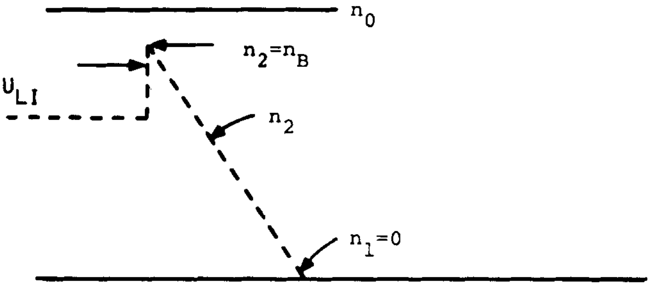


FIG. 17. Concentration profiles in Section I (mismatched velocities).

Section II is simply an extension of Section I. A similar strategy can be applied to Sections III and IV.

All the results derived up to this point still hold even if the carrier is present up to the discontinuities in Sections I and IV, rather than being confined to Sections II and III.

COMPARISONS BETWEEN MOVING AND STATIC BED PROCESSES

As was previously noted, the carrier requirements for a given production rate are the same in both processes.

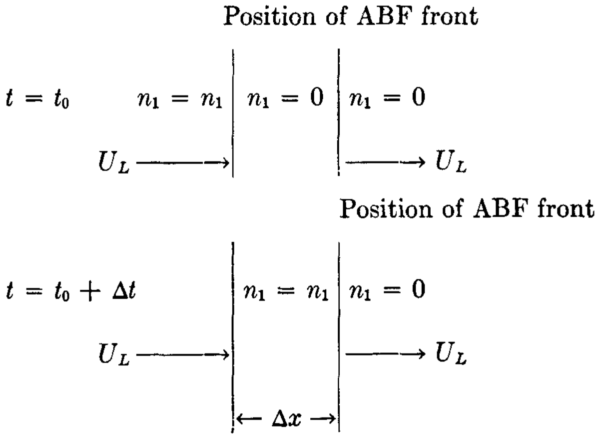
This equilibrium material balance model does not, however, provide any information on the inventory of packing needed in the moving bed process, and it shows that the inventory required in the static bed process can be made as small as desired by reducing the duration of the feed interval. Only when kinetic factors are introduced into both the static and moving bed systems is a comparison of packing inventory requirements possible.

Nevertheless, an analogy exists between the "solids-requirements" of the two processes. That is, the volume of solids passing through the countercurrent column per unit of product (R) is equal to the volume of the static bed column divided by the amount of feed in one injection:

$$R = (1/N_0)[(\alpha_{12}/(\alpha_{12} - 1)) - \phi] \tag{44}$$

APPENDIX A

Velocity of Front ABF



Input into Δx

$$n_1 \times U_L \Delta t$$

Output from Δx

$$0 \times U_L \Delta t$$

Accumulation

$$[(n_1 - 0) + (N_1 - 0)]\Delta x$$

Therefore

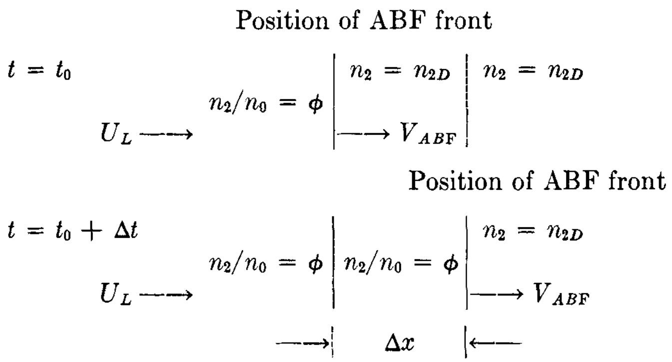
$$\Delta x/\Delta t = [U_L/1 + (N_1/n_1)]$$

but

$$\Delta x/\Delta t = V_{ABF}$$

APPENDIX B

Concentration of B(n_{2D}) between ABF and BCF Fronts:



By a material balance on the slice Δx, one obtains,

$$(n_{2F} - n_{2D})[1 + (N_{1F}/n_{1F})] = (N_{2F} + n_{2F}) - (N_{2D} + n_{2D})$$

Using Eqs. (1a) and (1c), one obtains the following implicit quadratic equation for n_{2D};

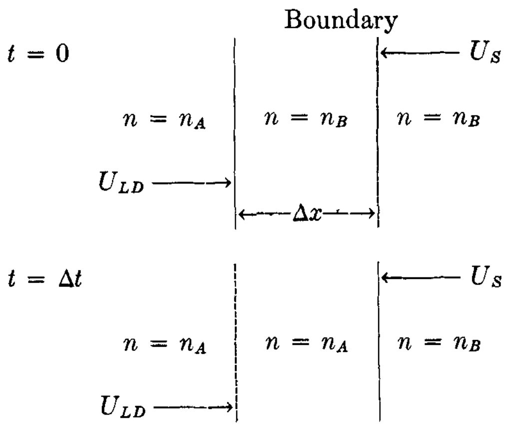
$$\frac{n_{2D}}{n_0}\left[\frac{\alpha_{23}}{1 + (\alpha_{23} - 1)(n_{2D}/n_0)} - \frac{\alpha_{12}}{1 + (\alpha_{12} - 1)(1 - \phi)}\right]$$

$$= \phi\left[\frac{1}{\alpha_{12} + (1 - \alpha_{12})\phi} - \frac{\alpha_{12}}{1 + (\alpha_{12} - 1)(1 - \phi)}\right]$$

Knowing the various separation factors and the feed composition, a solution for n_{2D} is obtained.

APPENDIX C

Moving Bed–Stationary Discontinuous Boundary



$$\text{input} = U_{LD}n_A + U_S N_B$$

$$\text{output} = U_S N_A + U_{LD}n_B$$

For stationary boundary the accumulation is zero, therefore

$$U_{LD}/U_S = (N_A - N_B)/(n_A - n_B)$$

since

$$\frac{N_A}{N_0} = \frac{\alpha(n_A/n_0)}{1 + (\alpha - 1)(n_A/n_0)}$$

and

$$\frac{N_B}{N_0} = \frac{\alpha(n_B/n_0)}{1 + (\alpha - 1)(n_B/n_0)}$$

Therefore,

$$\frac{U_{LD}}{U_S} = \frac{N_0}{n_0}\alpha\frac{1}{[1 + (\alpha - 1)(n_A/n_0)][1 + (\alpha - 1)(n_B/n_0)]}$$

Nomenclature

N_0	capacity of packing per column volume
N	concentration in packing per column volume

n_0	capacity of fluid per column volume
n	concentration in fluid per column volume
U	true velocity
x	distance from column inlet
t	time
L_M	minimum column length required for complete separation of A and B
t_c	time between end of one injection and the beginning of the next
R_D	ratio of carrier to feed
L	column length
R_C	ratio of column volume to rate of separation
Q_C	carrier leaving system
V	band velocity

Subscripts

1	strongly retained Component A
2	weakly retained Component B
3	intermediately retained Carrier C
I, II, III, IV	sections of chromatograph
S	column packing
L	carrier fluid

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